

FRAUDULENT CLAIMS AND NITPICKY INSURERS

Jean-Marc BOURGEON
Pierre PICARD

February 2012

Cahier n° 2012-06

DEPARTEMENT D'ECONOMIE

Route de Saclay

91128 PALAISEAU CEDEX

(33) 1 69333033

<http://www.economie.polytechnique.edu/>

<mailto:chantal.poujouly@polytechnique.edu>

Fraudulent Claims and Nitpicky Insurers

Jean-Marc Bourgeon* Pierre Picard†

February 22, 2012

Abstract

Insurance fraud is a major source of inefficiency in insurance markets. A self-justification of fraudulent behavior is that insurers are bad payers who start nitpicking if an opportunity arises, even in circumstances where the good-faith of policyholders is not in dispute. We relate this nitpicking activity to the inability of insurers to commit to their auditing strategy. Reducing the indemnity payments acts as an incentive device for the insurer since auditing is profitable even if the claim is not fraudulent. We show that optimal indemnity cuts are bounded above and that nitpicking remains optimal even if it induces adverse effects on policyholders' moral standards.

Keywords: Insurance Fraud, audit, no-commitment, nitpicking.

JEL Classification Numbers: D82, D86, G22.

*INRA and Ecole Polytechnique (Department of Economics). Address: UMR Économie Publique, 16 rue Claude Bernard, 75231 Paris Cedex 05, FRANCE. E-mail: bourgeon@agroparistech.fr

†Ecole Polytechnique (Department of Economics). Address: Department of Economics, Ecole Polytechnique, 91128 Palaiseau Cedex, FRANCE. E-mail: pierre.picard@polytechnique.edu

1 Introduction

Insurance fraud is widely considered to be a major source of inefficiency in insurance markets. However, although it is a recognized fact that fraud costs insurance companies billions of dollars every year,¹ it is striking to observe how insurance defrauders often do not perceive insurance claim padding as an unethical behavior and even tend to practice some kind of self-justification. A common view among consumers holds that insurance fraud would just be the rational response to the unfair behavior of insurance companies.² Consumers would tend to neutralize the psychological costs of their inappropriate behavior by considering it as the counterpart of the firms' unfair behavior: An eye for an eye would thus be the rule of the insurance fraud game (Strutton et al., 1994, 1997; Fukukawa et al., 2007).

Perceiving unfair behavior of insurance companies is often associated with the popular view according to which insurers would be bad payers that start nitpicking if an opportunity arises. Apart from the fact that disputes are sometimes induced by the deliberate bad faith of one of the two parties, more often than not consumers' complaints are motivated by the complexity of insurance contracts and by the difficulty to adapt oneself to (and even sometimes to figure out) all possible contingencies to which the contract may apply.³ It is true that insurance policies are usually very precise. They specify the various contingencies in which claims can be filed by policyholders, with exclusions and limits on payments. These clauses are often designed to lead policyholders to exert the appropriate effort when there is a risk of moral

¹According to the Coalition Against Insurance Fraud, insurance fraud steals at least \$80 billion every year in the US. See www.insurancefraud.org.

²Tennyson (1997, 2002) emphasizes that the psychological attitude toward insurance fraud is related to negative perceptions of insurance institutions. For instance, Tennyson (2002) shows that consumers who are not confident of the financial stability of their insurer and those who find auto insurance premiums to be burdensomely high are more likely than others to find fraud acceptable.

³An example, among thousands, drawn from The Telegraph (7 December 2011) illustrates how complexity may jeopardize the efficiency of risk sharing through insurance contracts: "A reader from Hampshire tells how she was caught out by the small print in her travel insurance policy when she took a taxi to Gatwick to catch a flight to Luxor. She was delayed by traffic and bad weather conditions and ended up missing her flight, and her holiday. Her insurer refused to pay out for the lost holiday (nearly £3,000), because the policy only specified cover travel by *scheduled public transport*. Insurance small print is one of the most common stumbling blocks for travelers and potentially the most expensive."

hazard. However, they also frequently allow insurers to reduce indemnity payment in circumstances where policyholders cannot be blamed for some deliberate inappropriate behavior.⁴ This may be at the origin of the feeling that the insurer legally profits from a situation where the policyholder is undoubtedly in good faith but the small print of the contract allows the insurer to deny the claim or to reduce the indemnity payment.

What is the logic of such behaviors? Why do insurers sometimes start nitpicking about claims, although the honesty of their customers is not disputed? This is a true puzzle because nitpicking induces some degree of uncertainty in the way the insurance contract is enforced, and for that reason it reduces the efficiency of the insurance coverage. Consequently, even if nitpicking is reflected in lower insurance premiums, the competition between insurers should lead them to offer the most efficient coverage and to refrain from such an apparently inefficient behavior. If nitpicking is so widespread in the insurance industry, its *raison d'être* must be related to insurance market mechanisms and not to the deviant behavior of some unscrupulous opportunistic insurers. The objective of this paper is to analyze this issue.

Our starting point is the behavior of insurers that are confronted with claims fraud. Insurers spend resources to monitor claims through a spectrum of verification procedures that go from the settling of the apparently honest claims in a routine way to the referral of most dubious claims to a Special Investigative Unit (SIU). Red flags and sometimes advanced scoring techniques may be used to channel claims in the most efficient way. The principles that guide these audit mechanisms have been analyzed in costly verification models where insureds have private information about their losses and insurers can verify claims by incurring an audit cost.⁵

Among various issues, this literature has shown how the ability of insurers to commit to an auditing policy may affect the efficiency of auditing mechanisms. In the most simple audit models, it is optimal to fully deter fraud through claims verification if insurers can commit to audit claims with a given probability whatever the hit rate. On the contrary, the fraud rate remains positive if insurers are unable to commit (see Picard, 1996). In more sophisticated models with heterogeneous policyholders, the impossibility to

⁴E.g., in cases where the clauses of the insurance contract are related neither with the origin of the accident nor with its severity and where the policyholder was not aware that his behavior would trigger the cancelation or a reduction in the indemnity payment.

⁵See Picard (2001) for a survey and Dionne et al. (2009) for an analysis of the link between claims auditing and the use of red flags, including scoring techniques.

commit induces a larger fraud rate than what would be optimal otherwise (see Dionne & Gagné, 2001). The fact that there is some positive fraud rate provides incentives to monitor claims, but this residual fraud will be reflected in higher insurance premiums paid by policyholders and ultimately by a less efficient risk sharing through insurance contracting.

Although the intensity of the commitment issue can be weakened in a dynamic setting where insurers can acquire the reputation of being tough auditors (Krawczyk, 2009) or when the monitoring of claims can be delegated to an independent agent in charge of investigating claims (Melumad & Mookherjee, 1989) or to a common agency (Picard, 1996), this commitment problem remains an issue. In what follows, we reconsider the commitment problem in a setting where the legal enforcement of insurance contracts gives some leeway to the insurer in deciding how much money to pay the claimant.

When insurers verify a claim, they can detect whether it is fraudulent or not. However, auditing claims may also provide information (a signal about the circumstances of an accident) that triggers some clauses of the contract that allow the insurer to partially or fully deny obligation to indemnify the policyholder. If moral hazard is not at stake, then conditioning the indemnity payment on such a random signal reduces the efficiency of the insurance coverage without exerting any incentive effect on policyholders. However, if the insurer cannot commit to the audit probability, then the possibility of exploiting such information obtained through audit acts as an incentive device for the insurer. To put it differently, if auditing may be profitable to the insurer even if the claim is not fraudulent, then he will be prompted to verify claim for a lower rate of fraud than if the only motive for auditing is to detect cheaters.

This issue can be reformulated in more general terms as an optimal contracting problem between principal and agent. The principal (the insurer) offers a risk-sharing contract to the agent (the policyholder). In a symmetric information setting, the optimal risk sharing involves a transfer that is a function of the random wealth to be shared between principal and agent and of nothing else. However, we know from Holmström (1979) that this conclusion is invalidated under moral hazard: the transfer from principal to agent should be a function not only of the wealth that has to be shared but also on any signal that would be informative on the agent's effort. In this paper we come to a similar conclusion in a setting without moral hazard: it is optimal to design the contract in such a way that the value of the signal affects the cost to the principal of making a decision (auditing claims with a

given probability) that would be *ex ante* mutually advantageous but to which the principal is unable to commit. Conditioning the transfer on the signal reduces the efficiency of the risk sharing agreement, but it also provides incentives to the principal, and ultimately a departure from the optimal risk sharing will be welfare improving.

From a more specific standpoint, our analysis is also related to the literature on “shrouded” costs that investigates the competition between firms that provide all-inclusive services, and firms that hide information on add-ups charged in addition to the price of a base service. In particular, Gabaix & Laibson (2006) show that shrouding firms survive at equilibrium when two types of consumers coexist on the market, rational ones that anticipate add-up costs and take advantage of low price of base services, and myopic ones. We also obtain that nitpicky firms survive the competition pressure, but in our framework there are no myopic consumers: they are all fully rational.

The paper is organized as follow. In Section 2, we investigate the nitpicking issue by considering a simple two-state setting where an accident corresponds to a unique loss level and insurers cannot commit to their audit strategy. We first show that it is optimal to adopt a nitpicking strategy that results in randomly cutting the contractual insurance indemnity on audited claims. We establish that nitpicking would be suboptimal if insurers were able to commit to their auditing strategy. Thus, the prevalence of nitpicking in insurance market reveals the existence of the commitment problem. We then characterize the optimal indemnity cut given the information gathered (i.e., the signal perceived) on each claim audited. We establish that the optimal indemnity cut is bounded above, meaning that the insurer refrains from reducing the indemnity payment in cases where he would be legally entitled to do so. Section 2 ends by investigating the effects of nitpicking on policyholders’ moral standards. Here we assume that defrauders incur a moral cost that decreases with the intensity of the insurer’s nitpicking activity. We show that despite this adverse effect on moral standards, nitpicking remains optimal. Section 3 extends our results to a more general setting in which losses may be more or less important. Section 4 investigates how nitpicking affects the incentives to exert an effort that diminishes the probability of an accident. We show that nitpicking used as a commitment device also allows the insurer to contain the moral hazard problem when it is not too intense. Section 5 concludes. All proofs are in Appendix.

2 The model

2.1 Nitpicking and optimal insurance contracts

Consider an insurance company providing coverage to individuals (households or businesses) against an accident that occurs with probability π and results in a loss L . Denote by P the insurance premium and I the contractual indemnity. Individuals file fraudulent claims with probability α . In other words, α denotes the probability to report a loss although no accident occurred. Thus, $\alpha(1 - \pi)/[\pi + \alpha(1 - \pi)]$ is the fraction of claims that are fraudulent.

The insurance company follows an audit rule which consists in verifying the genuineness of the accident for a proportion β of the claims. Let c be the cost of an audit, with $c < L$. Audit reveals without ambiguity a fraudulent claim, i.e., that no accident occurred (we discuss the case of an imperfect auditing in footnote 10), in which case the insured receives no indemnity and incurs a litigation cost B . If an accident did occur, auditing also provides detailed information about the circumstances of the accident and about the loss itself: which evidences the insured is able to provide about the extent of his loss, whether he did actually follow the requirements specified in the insurance contract, whether an exclusion applies, and so on. The insurance contract specifies cases in which the payment of the indemnity may be denied or reduced by the insurer even if the policyholder's good faith is not questioned. Hence, in case of an audit, the actual indemnity for an honest claim will be written as $(1 - \tilde{z})I$ where \tilde{z} corresponds to the fraction of the contractual reimbursement cut by the insurer. Such cuts vary case by case and thus \tilde{z} is a random variable. Circumstances that may trigger a cut in the indemnity payment are involuntary and do not correspond to a deliberate inappropriate behavior as in moral hazard issues: consequently \tilde{z} cannot be controlled by the policyholder. However, final decisions depend on the instructions received by claims handlers and the insurer somehow controls the intensity of his nitpicking activity, which is characterized by $q \in [0, 1]$. In a contract with nitpicking intensity q the insurance company save on average a fraction q of the contractual indemnity I . More precisely, the nitpicking technology is such that \tilde{z} is distributed according to the c.d.f. $F(z, q) = \Pr\{\tilde{z} \leq z|q\}$ over $[0, 1]$, where $q = E[\tilde{z}|q]$ and $\partial F/\partial q \leq 0$. Hence $q = 0$ corresponds to a deterministic insurance policy where the contractual indemnity I is paid without any restriction to all claimants when fraud has

not been detected. When $q > 0$, the indemnity payment is stochastic: a fraction \tilde{z} of the contractual indemnity I is retained and an increase in q shifts \tilde{z} in the sense of first-order stochastic dominance.

Here the nitpicking technology is taken as given and characterized by this family of probability distributions $F(\cdot, q)$ indexed by q . We will later show how an optimal nitpicking technology may be derived. In any case, we assume that nitpicking does not induce any additional cost and thus the insurer can commit to this behavior. Consequently, the nitpicking intensity q may be considered as the implicit part of the insurance contract, P and I being the explicit part.

Thus, the expected cost of claims (indemnity + audit cost) per policyholder is given by

$$C = [\pi(1 - \beta q) + (1 - \pi)(1 - \beta)\alpha]I + [\pi + (1 - \pi)\alpha]\beta c \quad (1)$$

The first term in (1) corresponds to the per-individual expected indemnity payment, where honest and fraudulent claims amount to outlays equal to $\pi(1 - \beta q)I$ and $(1 - \pi)(1 - \beta)\alpha I$ respectively. Note in particular that βq corresponds to the fraction of contractual indemnities not paid on average to honest claimants. The second term corresponds to the expected audit cost per policyholder: a contract results in an honest claim with probability π and a fraudulent one with probability $(1 - \pi)\alpha$, and claims are audited with probability β entailing a cost c .

Policyholders are risk averse. They maximize the expected utility $u(w_f)$ of final wealth w_f , with $u' > 0$, $u'' < 0$. The expected utility of an insured is written as

$$Eu = \pi\{(1 - \beta)u(w - P - L + I) + \beta E[u(w - P - L + (1 - \tilde{z})I)|q]\} \\ + (1 - \pi)\{(1 - \alpha)u(w - P) + \alpha[(1 - \beta)u(w - P + I) + \beta u(w - P - B)]\} \quad (2)$$

The first term in (2) corresponds to the expected utility in case an accident occurs: with probability $1 - \beta$ the insurance company pays the indemnity without auditing, while with probability β the claim is audited and the outcome is a partial payment of the contractual indemnity which depends on the insurer's nitpicking process. The second term corresponds to the no-accident situation, the insured being honest with probability $1 - \alpha$ or filing a fraudulent claim with probability α , in which case he receives the indemnity if no

audit occurs, but he faces litigation costs B if he is spotted.⁶

As a preliminary, let us first consider the benchmark case where insurers can commit to their auditing strategy. For policyholders to be deterred from filing fraudulent claims, Eu should be maximized at $\alpha = 0$, which requires $\beta \geq \beta^*(I, P)$ given by

$$\beta^*(I, P) = \frac{u(w - P + I) - u(w - P)}{u(w - P + I) - u(w - P - B)}. \quad (3)$$

Under a competitive insurance market, the optimal insurance contract $\{I, P, \beta, q\}$, including the preannounced audit probability β , maximizes the individuals' expected utility

$$V(I, P, \beta, q) \equiv (1 - \pi)u(w - P) + \pi u(w - P - L + I) - \pi\beta\{[u(w - P - L + I) - E[u(w - P - L + (1 - \tilde{z})I)|q]]\},$$

with respect to $I, P \geq 0, \beta, q \in [0, 1]$ under the non-negative profit constraint

$$P \geq \pi[I + \beta(c - qI)],$$

and

$$\beta \geq \beta^*(I, P).$$

The following proposition characterizes the optimal insurance contract in that case:

Proposition 1 *If the insurer can commit to its auditing strategy, then nit-picking is suboptimal: the optimal insurance contract is such that $q = 0$. In fact, if that were possible, the insurer should optimally reward honest insureds by awarding them a bonus R above the contractual indemnity I in case of an audit, and paying only I for non-audited claims.*

Proposition 1 shows that the insurer should choose $q = 0$ if it were able to commit to its auditing strategy whatever the fraud rate. Indeed, fraud would be fully deterred (i.e., $\beta = \beta^*(I, P)$ is optimal) and then nit-picking is suboptimal because it would artificially create an additional risk for the policyholder. This proposition also states that under the commitment

⁶We assume in this section that individuals do not feel social or moral pressure behaving dishonestly. We consider moral standards in Section 2.4.

assumption, payment should optimally be distorted in the other direction, by paying a higher indemnity $I + R$, with $R > 0$, for honest claims that are audited and only I otherwise.⁷ Nitpicking goes exactly in the opposite direction.

We now turn to the no-commitment case in the whole remaining part of this paper. In that case, the fraud strategy of the insured and the audit strategy of the insurer (α and β respectively) must be mutually best responses. As usual in audit models without commitment, insurers and policyholders play mixed strategies such that they are indifferent between the alternatives they may choose from (auditing or not, and defrauding or being honest respectively). For a given insurance contract (I, P, q) , using (1) and (2), one can easily verify that the Nash equilibrium of this insurer-policyholder game is such that $\beta = \beta^*(I, P)$ given by (3) and $\alpha = \alpha^*(I, q)$ given by

$$\alpha^*(I, q) = \frac{\pi(c - qI)}{(1 - \pi)(I - c)} \quad \text{if } c > qI, \quad (4)$$

and $\alpha^*(I, q) = 0$ if $c \leq qI$.

In particular, we may observe that the equilibrium fraud rate α^* is decreasing with respect to I and q , which shows that fraud may be decreased either by increasing the contractual indemnity or by nitpicking more intensely. If $c \leq qI$, then fraud fully vanishes at equilibrium, i.e., $\alpha^*(I, q) = 0$, because the return on nitpicking is larger than the audit cost, so that insurers will monitor claims even if there is no fraud. Hence, at equilibrium, either policyholders are indifferent between defrauding and being honest (when $c > qI$) or there is no fraud (when $c \leq qI$). In both cases, the policyholder's expected utility is written as

$$Eu^*(I, P, q) \equiv (1 - \pi)u(w - P) + \pi u(w - P - L + I) - \pi \beta^*(I, P) \{ [u(w - P - L + I) - E[u(w - P - L + (1 - \tilde{z})I) | q]] \}$$

⁷The intuition of this result is simple. At $R = 0$, slightly increasing R and decreasing I in such a way that the expected indemnity remains constant is welfare improving because such changes reduce the audit cost (from $\partial \beta^* / \partial I > 0$), with only second-order effect on expected utility. Note however that, in practice, it might be hard to implement such a distortion in favor of audited honest claimants. Indeed, announcing the proportion of audited claims would be necessary to sustain the insurer's commitment, but when $R > 0$ insurers would be better off hiding the number of verified claims to reduce aggregate indemnity payment. In other words, the bonus R may jeopardize the insurer's commitment to its auditing strategy.

Using $\beta = \beta^*(I, P)$ allows us to write the per-individual insurance cost as

$$C^*(I, q) \equiv [\pi + (1 - \pi)\alpha^*(I, q)]I = \pi(1 - q)I^2/(I - c).$$

The optimal insurance contract then solves

$$\max_{I, P, q} \{Eu^*(I, P, q) : P \geq C^*(I, q)\}. \quad (5)$$

Before moving to the core of our analysis, the following Lemma characterizes the optimal indemnity when there is no nitpicking.

Lemma 1 *If insurers are not allowed to cut the contractual reimbursement (i.e. if nitpicking is impossible: $q = 0$), then the optimal contract involves overinsurance: $I > L$.*

Lemma 1 states that the impossibility of insurers to commit to their auditing strategy leads them to offer insurance contracts with indemnity larger than loss (this was established by Boyer, 2004 in a slightly different model). The intuition is straightforward and is illustrated in Figure 1. Insurers are incited to audit claims if the expected gain of auditing is larger than the cost, which is the case if the fraud rate is larger than $\pi c/(1 - \pi)(I - c)$. If fraud were not at stake, then insurers would offer full coverage contract with $I = L$. Graphically, the optimal insurance contract is at the tangency point between the policyholder's indifference curve $Eu^*(I, P, 0) = u(w - \pi L)$ and the fair odds line $P = \pi I$. However, if the insured can defraud and insurers cannot commit to their auditing strategy, then the insurance premium is no longer given by $P = \pi I$ but by $P = C^*(I, 0)$. Slightly increasing I over L maintains the insurer's incentives at the right level for a lower fraud rate, and thus with a lower premium and only second-order risk-sharing effects, and ultimately that will be favorable to the insured. Thus, the tangency point between the zero profit line and the optimal indifference curve is above and on the right of the no-fraud case at $I = I^* > L$.⁸

The question is to determine if it is optimal that insurance companies nitpick claims when they are unable to commit to an audit strategy. Proposition 2 shows that this is actually the case.

⁸A similar result holds in principal-agent models with adverse selection and auditing but without risk sharing. For example, in a procurement problem with hidden cost, Khalil (1997) shows that contrary to the standard result of adverse selection problems, the optimal contract involves over-production for the high cost firm to induce the principal to audit the agent.

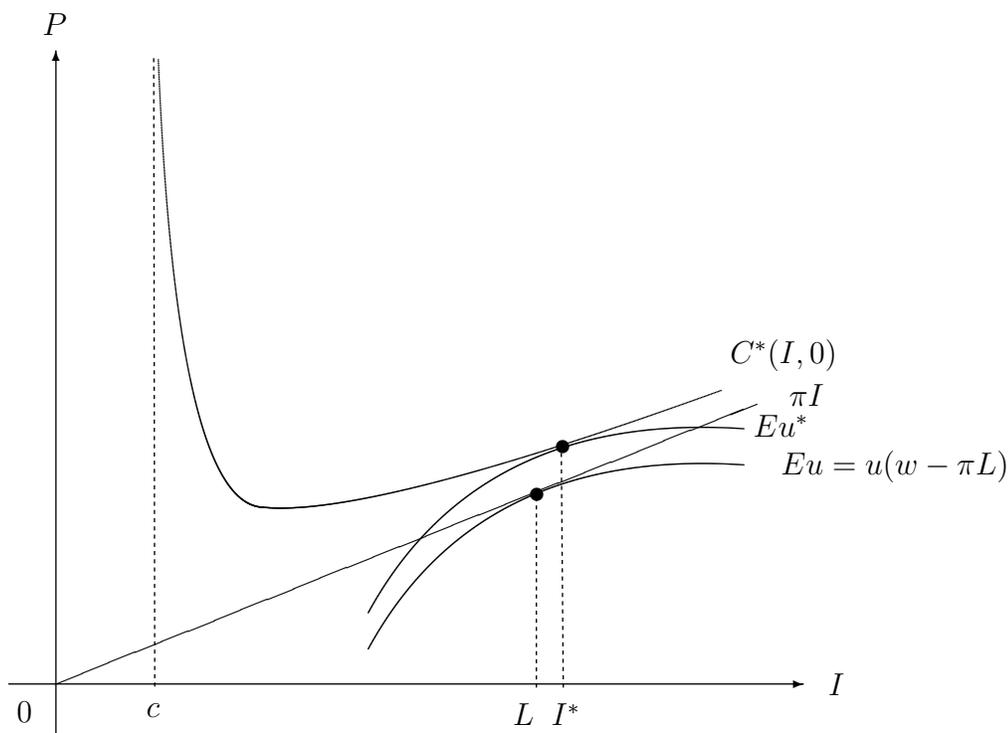


Figure 1: Optimal insurance contract without commitment.

Proposition 2 *The optimal insurance contract entails nitpicking, i.e. $q > 0$, even under the constraint $I \leq L$.*

Proposition 2 shows that optimal insurance contracting involves some degree of nitpicking by insurers. At first sight, this conclusion may seem paradoxical, since the optimal contract maximizes the policyholder's expected utility under the non-negative profit constraint. However, the intuition follows the same line as the one of Lemma 1. (4) shows that increasing q leads to a first-order decrease in the equilibrium fraud rate, because the gains drawn from nitpicking provide an additional incentives to monitor claims. Nitpicking also induces some degree of uncertainty in the insurance coverage, which reduces the attractiveness of the contract for policyholder. However, at the first order, the incentive effect dominates the risk sharing effect. Consequently, practicing some degree of nitpicking is favorable to the insured

himself. This conclusion still holds if overinsurance were prohibited, say because of a risk of moral hazard: the status quo situation without nitpicking would then be at $I = L$, with an unchanged conclusion.^{9,10}

2.2 Optimal nitpicking strategy

We have assumed so far that the nitpicking technology was given to the insurer, and that his only choice was the expected level of indemnity cut: i.e., the insurer just had to choose within a family of probability distributions $F(z, q)$ indexed by the nitpicking intensity q . In practice, a nitpicky insurer (in concrete terms a claims adjuster, possibly with the help of an expert or an investigator) may firstly obtain detailed information about the circumstances of the accident by auditing the claim. In a second stage, he has to decide whether or not and to what extent he will use this information to reduce the indemnity payment under the contractual indemnity although auditing didn't reveal any fraudulent behavior. At this second stage, the insurer may have some leeway when deciding on the indemnity allocated to the claimant.

Thus, we may consider that the insurer has to decide on indemnity cuts based on the information which is available to him after auditing. In what follows, this information is summarized by a random variable \tilde{x} distributed

⁹When $I > L$, an increase in q at $q = 0$ allows the insurer to operate a mean-preserving contraction in the policyholders' final wealth: the expected indemnity (and thus the premium) and the indemnity payment are reduced. When $I = L$, the risk induced by the reduction in the indemnity corresponds to a second-order welfare loss.

¹⁰Assuming imperfect auditing would not change our results but that would complicate the algebra. Suppose a fraction ε of fraudulent claims stay unspotted when audited. Following the same reasoning as above, we obtain that the equilibrium mixed strategies become

$$\alpha_\varepsilon(I, q) = \alpha^*(I, q)(I - c)/[(1 - \varepsilon)I - c] < \alpha^*(I, q)$$

and

$$\beta_\varepsilon^*(I, P) = \beta^*(I, P)/(1 - \varepsilon) > \beta^*(I, P).$$

Hence, the fraud rate threshold that makes the insurer indifferent between auditing or not auditing is reduced, while the audit rate threshold that makes no-loss individuals indifferent between filing a claim or being honest is increased. At equilibrium, the expected utility of the insured is similar to (3) but with $\beta^*(I, P)$ replaced by $\beta_\varepsilon^*(I, P)$ (and thus it decreases with ε) while the insurer per-individual cost becomes

$$C_\varepsilon(I, q) \equiv \pi(1 - q - \varepsilon)I^2/[(1 - \varepsilon)I - c]$$

which increases with ε .

over $[0, 1]$ with c.d.f. $G(x)$ and density $g(x) = G'(x)$. We interpret \tilde{x} as the maximum cut in the indemnity payment the insurer is legally entitled to apply, given all relevant available information on the claim. \tilde{x} cannot be controlled by the policyholder. It may correspond to accident circumstances or to loss estimates that are ambiguous, e.g., when there is no witness of a car accident, or when the policyholder cannot produce bills but only photos of property destroyed by fire, or when there is no second-hand market for used stolen goods... In such cases, the law of insurance contracts or decisions in court may specify whether an exclusion applies or how the loss should be valued, but the insurer has some leeway: he may either confine itself to strict legal stipulations or treat its customers with more consideration. Thus, the indemnity cut strategy z (i.e. the fraction of the contractual indemnity that will not be paid) is a function of x that must satisfy $z(x) \leq x$ for all x in $[0, 1]$.

It is straightforward to verify that the non-negative profit constraint is obtained by substituting $Ez(\tilde{x}) \equiv \int_0^1 z(x)dG(x)$ to q in function C^* . Now the policyholder's expected utility Eu^* depends on I, P and on function $z(\cdot)$ and it may be written as

$$Eu^* = (1 - \pi)u(w - P) + \pi[1 - \beta^*(I, P)]u(w - P - L + I) + \pi\beta^*(I, P) \int_0^1 u(w - P - L + (1 - z(x))I)dG(x).$$

Thus, still assuming a competitive insurance market, the optimal insurance contract, including the nitpicking strategy, solves

$$\max_{I, P, z(\cdot)} \{Eu^*(I, P, z(\cdot)) : P \geq C^*(I, Ez(\tilde{x})), 0 \leq z(x) \leq x \text{ for all } x\}.$$

The resulting nitpicking strategy is characterized is the following Proposition.

Proposition 3 *The optimal nitpicking strategy $z(\cdot)$ is characterized by a ceiling $\hat{x} > 0$ such that:*

$$z(x) = \begin{cases} x & \forall x < \hat{x} \\ \hat{x} & \forall x \geq \hat{x} \end{cases}$$

Proposition 3 states that the optimal strategy of the insurer is to reduce as much as possible the indemnity to what is legally possible up to ceiling \hat{x} . The intuition of this result is as follows. The optimal nitpicking strategy results

from a trade-off between on one hand the incentive advantage derived by the insurer from a more intense nitpicking activity, and on the other hand the negative effect of nitpicking on the risk coverage provided to policyholders. The incentive advantage of nitpicking depends on the average reduction in indemnity payment $Ez(\tilde{x})$. For a given average reduction $Ez(\tilde{x})$, the most efficient strategy in terms of risk sharing would involve a uniform percentage of reduction in the indemnity payment when a claim is audited. However, this is not a feasible strategy because when \tilde{x} is small, auditing may not provide enough relevant information to the insurer that would allow him to decrease the payment at the required level. Thus, the optimal strategy consists in reducing the payment as much as possible (i.e. with $z^*(x) = x$), but without creating too much disturbances in the risk coverage, hence the ceiling \hat{x} . In other words, if auditing has not revealed any fraudulent behavior, then the insurer should not exploit its information to decrease the indemnity over \hat{x} even if it could do so.

2.3 A case where fraud vanishes at equilibrium

The previous analysis shows that indemnity cut $z(\tilde{x})$ acts as a commitment device that prompts the insurer to audit claims. As shown in the definition of $\alpha^*(I, q)$ given by (4), the larger $q = Ez(\tilde{x})$, the lower the fraud rate that is necessary to preserve adequate audit incentives. The optimal contract trades off the drawback of less efficient risk coverage induced by indemnity cuts and the increase in insurance cost caused by fraud. As expressed by the constraint $z(\tilde{x}) \leq \tilde{x}$, the nitpicking intensity is constrained by the information that can be gathered through audit and also by the legal stipulations that put limits on the way the insurer can exploit this information to cut indemnities. In other words, the nitpicking strategy analyzed above corresponds to the optimal behavior of an insurer who is legally constrained in the way he determines the cut on contractual reimbursement. An interesting limit case is when there were no legal restraint on such cuts.¹¹ The insurer could then impose a unique cut, say q , on the contractual reimbursement of all audited claims, with $q \leq c/I$. When $q = c/I$ the indemnity cut is equal to the audit cost and thus insurers are willing to audit claims even if there are no fraudulent claims. The optimal contract would then be a solution to

$$\max_{I, P, q} \{Eu^*(I, P, q) : P \geq C^*(I, q), q \leq c/I\},$$

¹¹That would correspond to the case where $G(x) = 0$ for all $0 \leq x < 1$ and $G(1) = 1$.

with now

$$Eu^*(I, P, q) \equiv (1 - \pi)u(w - P) + \pi[1 - \beta^*(I, P)]u(w - P - L + I) + \pi\beta^*(I, P)u(w - P - L + (1 - q)I).$$

We show in the Appendix that :

Proposition 4 *If there is no legal restraint on the indemnity cut for audited claims, then the optimal contract is such that $q = c/I$ and no fraudulent claims are filed i.e., $\alpha = 0$.*

Proposition 4 highlights the strength of nitpicking: if no legal limit were imposed on indemnity cuts (or if these limits were not really constraining), then it would be more efficient to provide audit incentives through such an activity - which apparently goes against the policyholders' interest - than by tolerating a positive rate of fraud in the market.

2.4 Insurers' reputation and moral standards

Cutting unilaterally contractual indemnity for legitimate claims is certainly a cause of disputes and there is no doubt that it induces resentment against insurers. Thus, nitpicking could tarnish the insurers' reputation, with an adverse effect on policyholders' moral standards. Hence nitpicking may have the counterproductive result of inducing more individuals to file fraudulent claims.

To investigate this problem, consider that individuals incur moral costs when they defraud the insurers, and these costs depend on the insurers' reputation. The more insurers are known for nitpicking, the less the individuals' cost of defrauding them. Hence, when individuals behave honestly, they incur no moral cost, while their final wealth is reduced by $\gamma(q) > 0$ when they file a fraudulent claim, with $\gamma'(q) < 0$, where $q = Ez(\tilde{x})$. Compared to the previous sections, it is easily verified that only the audit probability (3) is affected by these costs. It now depends on q and becomes

$$\beta^*(I, P, q) = \frac{u(w - P - \gamma(q) + I) - u(w - P)}{u(w - P - \gamma(q) + I) - u(w - P - \gamma(q) - B)}, \quad (6)$$

with $\partial\beta^*/\partial q > 0$: the larger the nitpicking intensity, the larger the audit probability that dissuades policyholders from defrauding. Hence the optimal

contract is a solution to program (5) where the audit probability that affects the insured's expected utility Eu^* is $\beta^*(I, P, q)$ instead of $\beta^*(I, P)$. However, the fraud rate $\alpha^*(I, q)$ that induces auditing is independent from the link between the intensity of nitpicking and defrauders' moral costs. Thus, the logics of the reasoning that explained why nitpicking is optimal is not affected by the existence of such costs.

Proposition 5 *The optimal insurance contract still entails nitpicking, i.e. $q > 0$, when defrauders incur moral costs that decrease with q .*

We have observed that nitpicking would be a suboptimal behavior if insurers could commit to their audit strategy. This conclusion is still valid, and even reinforced if nitpicking induces lower moral standards. Indeed, when q is increasing from 0 to a positive value, the audit rate $\beta^*(I, P, q)$ that discourages fraud increases, and under the commitment assumption, this will induce a negative effect on the policyholder's equilibrium expected utility. This clear-cut difference between the commitment and no-commitment cases reinforces the idea that if nitpicking is a widespread phenomenon in the insurance industry despite its adverse effects on moral standards, that may be because it provides additional incentives to monitor claims, which is ultimately favorable to policyholders themselves.

3 Continuum of losses

Let us now consider a setting where accidents generate insurable losses that may differ from one claim to another. More explicitly, we assume that conditionally on the occurrence of the accident - which still occurs with probability π - the loss $\tilde{\ell}$ is a random variable distributed over an interval $(0, L]$ according to the c.d.f. $F(\ell)$ with density $f(\ell) = F'(\ell)$. In that case, there are two reasons for the insurer to audit claims: firstly to verify the occurrence of the accident and secondly to assess the extent of the loss compared to the insured's claim. So, in addition to detect claims that do not correspond to a true accident, the audit of claims also aims at spotting the individuals who build up their claims. In practice, the cost of auditing claims is increasing with the size of the claim. For simplicity, we will assume that this cost is proportional to the announced loss i.e., it is given by $c\ell$ for claim ℓ , with $0 < c < 1$.

The further developments are guided by the following intuition. Because losses differ between claims, individuals' incentives to overstate losses also differ: an individual having experienced a large loss expects to receive a large indemnity from the insurer if his claim is honest. Relative to this indemnity, an overstatement of a given size of his loss would only lead to a relatively small increase in the indemnity if the claim is not audited, or to a reimbursement corresponding to the insurer's assessment of the loss and litigation costs in case of an audit. Moreover, the policyholder may expect a larger probability to have his claim audited if he announces a larger loss.

Let $EU(\ell, \hat{\ell})$ be the expected utility of an individual with loss ℓ who overstates his loss by announcing $\hat{\ell}$ larger than ℓ and let $EU(\ell)$ be the expected utility in the case of an honest claim $\hat{\ell} = \ell$. The audit probability may depend on the size of the claim: it will be denoted by $\beta(\hat{\ell})$ for a claim $\hat{\ell}$. The indemnity actually paid is now a function $I(\cdot)$ of the claim $\hat{\ell}$ if there is no audit, and of the true loss, possibly reduced by the nitpicking activity of the insurer, if the claim is audited. In this second case, the indemnity payment is $I((1 - z(\tilde{x}))\ell)$, where \tilde{x} is defined as before and $z(\tilde{x})$ is the reduction rate in the assessment of the claim, with $z(x) \leq x$ for all x in $[0, 1]$. We simplify matters by restricting attention to insurance contracts without overinsurance, possibly because overinsurance would induce adverse consequences in terms of moral hazard. We thus assume $0 \leq I(\ell) \leq \ell$ for all ℓ . The policyholder still incurs the litigation cost B if auditing reveals fraud and, for simplicity, we here consider the case where individuals do not have moral costs of defrauding. Thus, we may write

$$EU(\ell, \hat{\ell}) \equiv \beta(\hat{\ell})Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) - B) + [1 - \beta(\hat{\ell})]u(w - P - \ell + I(\hat{\ell}))$$

if $\hat{\ell} > \ell$ and

$$EU(\ell) \equiv \beta(\ell)Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell)) + [1 - \beta(\ell)]u(w - P - \ell + I(\ell)).$$

An individual with loss $\ell > 0$ tells the truth (i.e. announces $\hat{\ell} = \ell$) if the incentive constraints $EU(\ell) \geq EU(\ell, \hat{\ell})$ are satisfied for all $\hat{\ell}$ in $(\ell, L]$. For individuals who did not experience an accident, the incentives constraints are written as

$$u(w - P) \geq \beta(\hat{\ell})u(w - P - B) + [1 - \beta(\hat{\ell})]u(w - P + I(\hat{\ell})) \quad (7)$$

for all $\hat{\ell}$ in $(0, L]$. Hence, for these individuals, defrauding is equivalent to choosing one of the lotteries $Z(\hat{\ell}) \equiv \{-B, \beta(\hat{\ell}); I(\hat{\ell}), 1 - \beta(\hat{\ell})\}$, $\hat{\ell} \in (0, L]$,

and their incentive constraints are satisfied if these insured, with status quo wealth $w - P$, do not benefit from choosing one of these lotteries $Z(\hat{\ell})$.

Lemma 2 *If individuals display NIARA preferences¹² and if $I(\ell)$ and $\beta(\ell)$ are non-decreasing in ℓ , then all policyholders with losses $\ell > 0$ are deterred to file a fraudulent claim if the incentive constraints (7) of the individuals with no loss are satisfied.*

The intuition of Lemma 2 is simple. For a type ℓ individual (with $\ell = 0$ if he has not experienced any accident), filing a fraudulent claim $\hat{\ell}$ larger than ℓ is a risky choice: he gains $I(\hat{\ell}) - I(\ell)$ if his claim is not audited, but he has to pay B in case of an audit. Note that when $\ell > 0$, the earnings in case of truthful revelation of the loss may also be uncertain in case of nitpicking. However, under the assumptions made in the Lemma, the wealth of an individual who does not defraud is always larger when $\ell = 0$ than when $\ell > 0$. If the individuals' absolute risk aversion is not increasing with wealth and if policyholders are deterred to file fraudulent claims (which is a risky venture) when $\ell = 0$, then *a fortiori* they will choose not to defraud when $\ell > 0$.

We assume in the following that the optimal insurance contract satisfies the condition of Lemma 2. Of course this will have to be checked. Incentive constraints are thus satisfied for all ℓ in $(0, L]$ if they are satisfied for $\ell = 0$. The optimal audit strategy is such that (7) is binding for all $\ell > 0$, leading no-loss individuals to be indifferent between being honest or announcing any claim $\ell \in (0, L]$. The audit probability written as a function $\beta(\ell)$ of the claim size thus satisfies

$$\beta(\ell) = \frac{u(w - P + I(\ell)) - u(w - P)}{u(w - P + I(\ell)) - u(w - P - B)}. \quad (8)$$

Under the assumptions made in Lemma 2, we may consider the case where only no-loss individuals file fraudulent claims. The expected cost $C(\ell)$ of the claims of size ℓ per policyholder then satisfies

$$\begin{aligned} C(\ell)h(\ell) = & \pi\{\beta(\ell)[c\ell + EI((1 - z(\tilde{x}))\ell)] + [1 - \beta(\ell)]I(\ell)\}f(\ell) \\ & + (1 - \pi)\sigma p(\ell)\{\beta(\ell)c\ell + [1 - \beta(\ell)]I(\ell)\} \end{aligned}$$

¹²Non Increasing Absolute Risk Aversion.

where $h(\ell)$ is the density function of the size of the claims, σ is the probability that a no-loss individual decides to defraud the insurer, and $p(\ell)$ is the density of the size of fraudulent claims over $(0, L]$. In other terms, a defrauder (necessarily a no-loss individual) chooses to announce $\hat{\ell} \in [\ell, \ell + d\ell]$ with probability $p(\ell)d\ell$ and claims will be in the same interval with probability $h(\ell)d\ell$. The insurer is indifferent between auditing those claims or not auditing if

$$p(\ell) = \frac{\pi\{EI((1 - z(\tilde{x}))\ell) + c\ell - I(\ell)\}f(\ell)}{\sigma(1 - \pi)[I(\ell) - c\ell]}$$

We must have $\int_0^L p(\ell)d\ell = 1$ for $p(\cdot)$ to be a density function, which yields the fraud probability of no-loss individuals

$$\sigma = \frac{\pi}{1 - \pi} \left\{ \int_0^L \frac{EI((1 - z(\tilde{x}))\ell)}{I(\ell) - c\ell} f(\ell)d\ell - 1 \right\}.$$

The claims probability distribution then satisfies

$$\begin{aligned} h(\ell) &= \pi f(\ell) + (1 - \pi)\sigma p(\ell) \\ &= \pi f(\ell) \frac{EI((1 - z(\tilde{x}))\ell)}{I(\ell) - c\ell} \end{aligned}$$

At equilibrium, $C(\ell) = \pi I(\ell)$ and the expected cost of a contract for the insurer simplifies to

$$C = \int_0^L \pi I(\ell)h(\ell)d\ell = \pi E \left[\frac{I(\ell)I((1 - z(\tilde{x}))\tilde{\ell})}{I(\tilde{\ell}) - c\tilde{\ell}} \right].$$

The expected utility of the insured is given by

$$\begin{aligned} Eu^*(I(\cdot), P, z(\cdot)) &\equiv (1 - \pi)u(w - P) + \pi Eu(w - P - \tilde{\ell} + I(\tilde{\ell})) \\ &\quad - \pi E[\beta(\tilde{\ell})\{u(w - P - \tilde{\ell} + I(\tilde{\ell})) - u(w - P - \tilde{\ell} + I((1 - z(\tilde{x}))\tilde{\ell}))\}] \end{aligned}$$

Assuming a competitive insurance market, the optimal contract satisfies

$$\max_{I(\cdot), z(\cdot), P} \left\{ Eu^*(I(\cdot), P, z(\cdot)) : P \geq \pi E \left[\frac{I(\tilde{\ell})I((1 - z(\tilde{x}))\tilde{\ell})}{I(\tilde{\ell}) - c\tilde{\ell}} \right], 0 \leq I(\tilde{\ell}) \leq \tilde{\ell} \right\} \quad (9)$$

where $\beta(\cdot)$ is given by (8) and where expectation is taken with respect to \tilde{x} and $\tilde{\ell}$. Program (9) extends the problem considered in Section 2.2 to the case where losses may have different sizes, with a similar conclusion :

Proposition 6 *When individuals incur losses of different size, the optimal insurance contract entails some degree of nitpicking by insurers: $Ez(\tilde{x}) > 0$.*

4 Ex ante moral hazard and nitpicking

We argued above that the small print of an insurance contract may reduce the policyholders' moral hazard. Indeed, Holmström (1979) shows that under moral hazard, it is optimal to condition the indemnity on any information that is informative on the policyholder's effort. Thus, in the setting of this paper, if the probability distribution of \tilde{x} were affected by the policyholder's effort, then designing an insurance contract with a contingent indemnity would serve at the same time to reduce the (*ex ante*) moral hazard and to fight the (*ex post*) claims fraud.

Consider that the probability of the loss - of a given size L - depends on the behavior of the policyholder: it is denoted by π_e where e is the policyholder's effort. For the sake of simplicity, assume there are only two levels of effort: $e \in \{0, 1\}$ and $\pi_0 > \pi_1$: Thus, the probability of the accident is diminished if the policyholder exerts an effort $e = 1$ rather than no effort $e = 0$. Let d_e be the desutility of exerting effort e , with $d_1 > d_0$. The policyholder's behavior is not observable by the insurer, but it is related to the verifiable index $\tilde{x} \in [0, 1]$. Let $G_e(x) = \Pr\{\tilde{x} \leq x|e\}$ denote the c.d.f. of \tilde{x} conditional on e . A decrease of e from 1 to 0 shifts the distribution of \tilde{x} in the sense of first-order stochastic dominance, i.e., $G_0(x) \leq G_1(x)$ for all x in $[0, 1]$, with a strong inequality on a positive measure subset. When the insurer audits a claim, he observes if the accident did actually occur or not and he also observes the realization of \tilde{x} . Because the policyholder's utility is separable between final wealth and effort desutility, the policyholder's incentive to defraud does not depend on the effort undertaken *ex ante*: *ex post*, all individuals who didn't suffer any loss face the same dilemma when considering filing a fraudulent claim. The policyholder's expected utility with contract $\{I, P, z(\cdot)\}$ and effort e thus simplifies to $Eu_e^* - d_e$ where

$$\begin{aligned} Eu_e^* &= (1 - \pi_e)u(w - P) + \pi_e[1 - \beta^*(I, P)]u(w - P - L + I) \\ &\quad + \pi_e\beta^*(I, P) \int_0^1 u(w - P - L + (1 - z(x))I) dG_e(x) \end{aligned}$$

and he is induced to exert an effort $e = 1$ if $Eu_1^* - Eu_0^* \geq d_1 - d_0$.

Proposition 7 *Under the optimal insurance contract $\{P, I, z(\cdot)\}$ characterized in Proposition 3, policyholders are induced to exert the high level of effort $e = 1$ provided that $\pi_0 - \pi_1$ and $d_1 - d_0$ are not too large.*

Thus, when the moral hazard problem is not too intense, nitpicking claims with the only objective of providing audit incentives induce individuals to exert the high level of effort. Contrary to deductibles, indemnity cuts only affect audited claims and may vary from case to case depending on the information gathered during the audit.

5 Conclusion

Thus, nitpicking may prove to be welfare improving in insurance markets. There are indeed (at least) two justifications for cutting down the indemnities on the basis of information collected through claims auditing: solving the *ex ante* moral hazard problem when perceived signals are informative on the policyholder's effort, and improving the insurers' *ex post* commitment to audit claims. When the focus is on this commitment problem, optimal nitpicking trades off the drawback of a less favorable risk sharing between insurer and insured against the efficiency gain of a lower equilibrium fraud rate. Policyholders may legitimately complain of insurers' unfair behavior in the sense that nitpicking induces some degree of horizontal inequality. However, this behavior is the rational response to a commitment problem, and ultimately improving the credibility of the claim's verification strategy is in the policyholders' best interest. Nitpicking is nevertheless a second-best strategy: if insurers could make their claims monitoring perfectly credible, for instance through a reputation effect or by delegating audit to independent agents, then nitpicking would become a suboptimal strategy. In that sense, nitpicking reveals the failure of these commitment devices.

Several extensions of our analysis would be worth investigating. In Section 3, we have considered the case of losses that may differ from one individual to the other. However, we have restricted attention to a setting where at equilibrium the only defrauders are individuals who have not experienced an accident, while the policyholders who suffered a loss are deterred to build up their claims. As loss overstatement is commonly found in insurance markets, it would be interesting to extend our framework to a setting where claims build up actually occurs at equilibrium. In Section 4, we have shown that

nitpicking induces individuals to exert an effort that diminishes the probability of the loss when the cost of this prevention effort is not too large. If the moral hazard problem is more intense, then insurers should face the commitment problem and the moral hazard problem simultaneously. Interesting questions then arise regarding the design of the indemnity schedule and the nitpicking intensity. For instance, Proposition 3 has shown that the optimal strategy is characterized by an upper limit on indemnity cuts. This may not be true anymore under moral hazard if the law of insurance contracts allows the insurers to fully cancel indemnity payments in cases that reveal a likely inappropriate behavior of the policyholder.

References

- Boyer, M. M. (2004). Overcompensation as a partial solution to commitment and renegotiation problems: The case of ex post moral hazard. *Journal of Risk and Insurance*, 71(4), 559–582.
- Dionne, G. & Gagné, R. (2001). Deductible contracts against fraudulent claims: Evidence from automobile insurance. *Review of Economics and Statistics*, 83(2), 290–301.
- Dionne, G., Giuliano, F., & Picard, P. (2009). Optimal auditing with scoring: Theory and application to insurance fraud. *Management Science*, 55, 58–70.
- Fukukawa, K., Ennew, C., & Diacon, S. (2007). An eye for an eye: Investigating the impact of consumer perception of corporate unfairness on aberrant consumer behavior. In P. Flanagan, P. Primeaux, & W. Ferguson (Eds.), *Insurance Ethics for a More Ethical World (Research in Ethical Issues in Organizations)*, volume 7 (pp. 187–221). Emerald Group Publishing Limited.
- Gabaix, X. & Laibson, D. (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, 121(2), 505–540.
- Holmström, B. (1979). Moral hazard and observability. *Bell Journal of Economics*, 10(1), 79–91.
- Khalil, F. (1997). Auditing without commitment. *The RAND Journal of Economics*, 28(4), 629–640.
- Krawczyk, M. (2009). The role of repetition and observability in deterring insurance fraud. *The Geneva Risk and Insurance Review*, 34, 77–87.
- Melumad, N. D. & Mookherjee, D. (1989). Delegation as commitment: The case of income tax audits. *The RAND Journal of Economics*, 20(2), pp. 139–163.
- Picard, P. (1996). Auditing claims in insurance market with frauds: the credibility issue. *Journal of Public Economics*, 63, 27–56.

- Picard, P. (2001). Economic analysis of insurance fraud. In G. Dionne (Ed.), *Handbook of Insurance*, volume 22 of *Huebner International Series on Risk, Insurance and Economic Security* (pp. 315–362). Springer Netherlands.
- Strutton, D., Pelton, L. E., & Ferrell, O. C. (1997). Ethical behavior in retail settings: Is there a generation gap? *Journal of Business Ethics*, 16, 87–105.
- Strutton, D., Vitell, S. J., & Pelton, L. E. (1994). How consumers may justify inappropriate behavior in market settings: An application on the techniques of neutralization. *Journal of Business Research*, 30(3), 253 – 260.
- Tennyson, S. (1997). Economic institutions and individual ethics: A study of consumer attitudes toward insurance fraud. *Journal of Economic Behavior & Organization*, 32(2), 247 – 265.
- Tennyson, S. (2002). Insurance experience and consumers' attitudes toward insurance fraud. *Journal of Insurance Regulation*, 21(2), 35–55.

Appendix

A Proof of proposition 1

Let $\{I, P, \beta, q\}$ be the optimal contract when the insurer can commit and suppose $q > 0$. Consider the alternative contract $\{\hat{I}, \hat{P}, \hat{\beta}, \hat{q}\}$ with $\hat{P} = P, \hat{\beta} = \beta, \hat{q} = 0$ and $\hat{I} = I(1 - \beta q)$. We have: $\hat{P} = P \geq \pi[I + \beta(c - qI)] = \pi(\hat{I} + c\hat{\beta})$. Furthermore, using $\partial\beta^*/\partial I > 0, I > \hat{I}$ and $P = \hat{P}$ gives $\hat{\beta} = \beta \geq \beta^*(I, P) > \beta^*(\hat{I}, \hat{P})$. Hence $\{\hat{I}, \hat{P}, \hat{\beta}, \hat{q}\}$ is feasible. Using the concavity of $u(\cdot)$ yields

$$\begin{aligned} V(I, P, \beta, q) &\equiv \pi\{[1 - \beta]u(w - P - L + I) + \beta E[u(w - P - L + (1 - \tilde{z})I)|q]\} \\ &\quad + (1 - \pi)u(w - P) \\ &< \pi u(w - P - L + I - \beta qI) + (1 - \pi)u(w - P) \\ &= \pi u(w - \hat{P} - L + \hat{I}) + (1 - \pi)u(w - \hat{P}) = V(\hat{I}, \hat{P}, \hat{\beta}, \hat{q}), \end{aligned}$$

which contradicts the optimality of $\{I, P, \beta, q\}$.

Let us now prove that it is optimal to grant a bonus $R > 0$ to non-fraudulent audited claims. In that case, the policyholder's expected utility becomes

$$W(I, R, P, \beta) \equiv \pi[(1 - \beta)u(w - P - L + I) + \beta u(w - P - L + I + R)] + (1 - \pi)u(w - P),$$

while the non-negative profit constraint is now written as $P \geq \pi[I + \beta(c + R)]$. The optimal contract with commitment thus solves

$$\max_{P, I, R, \beta} \{W(I, R, P, \beta) : P \geq \pi[I + \beta(c + R)], \beta \geq \beta^*(I, P)\}.$$

Denote by $\mathcal{L}(I, R, P, \beta, \lambda, \mu) \equiv W(I, R, P, \beta) + \lambda\{P - \pi[I + \beta(c + R)]\} + \mu[\beta - \beta^*(I, P)]$ the Lagrangian of this program. The first-order conditions with respect to I, R and β are written as

$$\partial\mathcal{L}/\partial I = \pi[(1 - \beta)u'(w - P - L + I) + \beta u'(w - P - L + I + R)] - \lambda\pi - \mu\partial\beta^*/\partial I = 0, \quad (10)$$

$$\partial\mathcal{L}/\partial\beta = -\pi[u(w - P - L + I) - u(w - P - L + I + R)] - \lambda\pi(c + R) + \mu = 0, \quad (11)$$

and

$$\partial\mathcal{L}/\partial R = \pi\beta[u'(w - P - L + I + R) - \lambda] \leq 0, \quad (12)$$

with an equality if $R > 0$. Suppose $R = 0$ at the optimum. Using (11), we get $\mu = \lambda\pi c$ while (10) and $\partial\beta^*/\partial I > 0$ yield

$$u'(w - P - L + I) = \lambda(1 + c\partial\beta^*/\partial I) > \lambda,$$

which contradicts (12). ■

B Proof of Lemma 1

Let $\mathcal{L}(I, P, q, \lambda) \equiv Eu^*(I, P, q) + \lambda[P - C^*(I, q)]$ be the Lagrangian of program (5), and let $I^*(q), P^*(q)$ and $\lambda^*(q)$ be the optimal indemnity, premium and Lagrange multiplier for a given nitpicking intensity q . We get

$$(\partial\mathcal{L}/\partial P)|_{q=0} = -[(1-\pi)u'(w-P^*(0)) + \pi u'(w-P^*(0)-L+I^*(0))] + \lambda^*(0) = 0,$$

$$(\partial\mathcal{L}/\partial I)|_{q=0} = \pi u'(w-P^*(0)-L+I^*(0)) - \lambda^*(0)\pi(I^*(0)-2c)I^*(0)/(I^*(0)-c)^2 = 0,$$

and, as $(I-c)^2 > I^2 - 2cI$,

$$\lambda^*(0) = (1-\pi)u'(w-P^*(0)) + \pi u'(w-P^*(0)-L+I^*(0)) > u'(w-P^*(0)-L+I^*(0)),$$

which implies

$$u'(w - P^*(0)) > u'(w - P^*(0) - L + I^*(0)),$$

hence $I^*(0) > L$ from $u'' < 0$.

C Proof of Proposition 2

Denoting $\psi(q)$ be the value function of Problem 5 for a given value of q . The Envelop Theorem gives

$$\begin{aligned} \psi'(q) &= \pi\beta^*(I^*(q), P^*(q)) \frac{\partial E}{\partial q} [u(w - P - L + (1 - \tilde{z})I)|q] \Big|_{I=I^*(q), P=P^*(q)} \\ &\quad + \frac{\lambda^*(q)\pi I^*(q)^2}{I^*(q) - c}. \end{aligned}$$

Integrating by part allows us to write

$$E[u(w - P - L + (1 - \tilde{z})I)|q] = u(w - P - L) + I \int_0^1 u'(w - P - L + (1 - z)I) F(z, q) dz,$$

and thus

$$\begin{aligned} & \frac{\partial E}{\partial q} [u(w - P - L + (1 - \tilde{z})I) | q] \Big|_{I=I^*(q), P^*(q)} \\ &= I^*(q) \int_0^1 u'(w - P^*(q) - L + (1 - z)I^*(q)) \frac{\partial F(z, q)}{\partial q} dz. \end{aligned}$$

Hence, from the mean-value theorem there exists $\hat{z}(q) \in (0, 1)$ such that

$$\begin{aligned} & \frac{\partial E}{\partial q} [u(w - P - L + (1 - \tilde{z})I) | q] \Big|_{I=I^*(q), P^*(q)} \\ &= I^*(q) u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)) \int_0^1 \frac{\partial F(z, q)}{\partial q} dz \\ &= -I^*(q) u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)), \end{aligned}$$

where the last equality is obtained by differentiating the identity $q = E[\tilde{z} | q] = 1 - \int_0^1 F(z, q) dz$, which gives $\int_0^1 \partial F(z, q) / \partial q dz = -1$. As $\beta^* < 1$, we have

$$\psi'(q) > \pi [-I^*(q) u'(w - P^*(q) - L + (1 - \hat{z}(q))I^*(q)) + \lambda^*(q) I^*(q)^2 / (I^*(q) - c)],$$

and finally, using $\hat{z}(0) = 0$ and $u'(w - P^*(0) - L + I^*(0)) < \lambda^*(0)$, we get

$$\psi'(0) > \pi \lambda^*(0) I^*(0) [-1 + I^*(0) / (I^*(0) - c)] = \pi c \lambda^*(0) I^*(0) / (I^*(0) - c) > 0,$$

which implies $q > 0$ at an optimal solution to Problem 5. ■

D Proof of proposition 3

Pointwise maximization with respect to $z(x)$ yields

$$[-\pi \beta^* I u'(w - P - L + (1 - z(x))I) + \lambda \pi I^2 / (I - c)] g(x) \geq 0, \quad (13)$$

for all $x \in [0, 1]$ with an equality if $z(x) < x$. Consequently, for all x such that $g(x) > 0$ we have $\beta^* u'(w - P - L + (1 - z(x))I) = \lambda I / (I - c)$ when $z(x) < x$, and $\beta^* u'(w - P - L + (1 - z(x))I) \leq \lambda I / (I - c)$ when $z(x) = x$. As $u(\cdot)$ is concave, $u'(w - P - L + (1 - z)I)$ is strictly increasing in z , implying that the optimal solution is characterized as in Proposition 3, with threshold \hat{x} defined by

$$\beta^* u'(w - P - L + (1 - \hat{x})I) = \lambda I / (I - c),$$

for optimal values of P, I and λ . ■

E Proof of proposition 4

Denote by $\mathcal{L}(I, P, q, \lambda) \equiv Eu^*(I, P, q) + \lambda[P - C^*(I, q)]$ the Lagrangian of the insurer's program and assume that $q < c/I$ at the optimum. The first-order conditions with respect to I and q are given by

$$\begin{aligned} \partial\mathcal{L}/\partial I &= \pi[(1 - \beta^*)u'(w - P - L + I) + (1 - q)\beta^*u'(w - P - L + (1 - q)I)] \\ &\quad - (\partial\beta^*/\partial I)[u(w - P - L + I) - u(w - P - L + (1 - q)I)] \\ &\quad - \lambda(1 - q)I(I - 2c)/(I - c)^2 = 0, \end{aligned} \quad (14)$$

and

$$\partial\mathcal{L}/\partial q = \pi I[-\beta^*u'(w - P - L + (1 - q)I) + \lambda I/(I - c)] = 0. \quad (15)$$

Rearranging terms of (14) and using (15) yield

$$\begin{aligned} (1 - \beta^*)u'(w - P - L + I) &= \frac{\partial\beta^*}{\partial I}[u(w - P - L + I) - u(w - P - L + (1 - q)I)] \\ &\quad - \lambda \frac{(1 - q)Ic}{(I - c)^2}, \end{aligned}$$

where

$$\frac{\partial\beta^*}{\partial I} = \frac{u'(w - P + I)[u(w - P) - u(w - P - B)]}{[u(w - P + I) - u(w - P - B)]^2}.$$

Using the concavity of $u(\cdot)$ yields

$$u(w - P + I) - u(w - P) > u'(w - P + I)I,$$

and, using (15) and the concavity of $u(\cdot)$ again, we may write

$$\begin{aligned} u(w - P - L + I) - u(w - P - L + (1 - q)I) &< qIu'(w - P - L + (1 - q)I) \\ &= q\lambda I^2/[(I - c)\pi\beta^*]. \end{aligned}$$

Consequently

$$\begin{aligned} &\frac{\partial\beta^*}{\partial I}[u(w - P - L + I) - u(w - P - L + (1 - q)I)] \\ &< \frac{q\lambda I^2}{(I - c)} \times \frac{u'(w - P + I)[u(w - P) - u(w - P - B)]}{[u(w - P + I) - u(w - P - B)][u(w - P + I) - u(w - P)]} \\ &< \frac{q\lambda I}{\pi(I - c)} \times \frac{u(w - P) - u(w - P - B)}{u(w - P + I) - u(w - P - B)} < \frac{q\lambda I}{\pi(I - c)}. \end{aligned}$$

We thus have

$$(1 - \beta^*)u'(w - P - L + I) < \frac{q\lambda I}{I - c} - \frac{\lambda(1 - q)Ic}{(I - c)^2} = \lambda I \frac{qI - c}{(I - c)^2} < 0,$$

which contradicts $\beta^* < 1$. We thus have $q = c/I$ and thus $\alpha = \alpha^*(I, c/I) = 0$.

■

F Proof of proposition 5

For the sake of brevity, the proof is made in the case of a given nitpicking technology as in Proposition 2. It extends straightforwardly to an optimal nitpicking strategy as defined in Proposition 3. Let $\psi(q)$ be the value function of Problem 5, with $\beta^* = \beta^*(I, P, q)$ given by (6) instead of $\beta^* = \beta^*(I, P)$. The Envelope Theorem gives

$$\begin{aligned} \psi'(q) &= \pi \beta^*(I, P, q) \frac{\partial E}{\partial q} [u(w - P - L + (1 - \tilde{z})I) | q] \Big|_{I=I^*(q), P=P^*(q)} + \frac{\lambda^*(q) \pi I^*(q)^2}{I^*(q) - c} \\ &- \pi \frac{\partial \beta^*(I, P, q)}{\partial q} \Big|_{I=I^*(q), P=P^*(q)} \{u(w - P^*(q) - L + I^*(q)) - E[u(w - P - L + (1 - \tilde{z})I) | q]\}. \end{aligned}$$

where the last bracketed term tends to 0 when $q \rightarrow 0$. As a result, we have $\psi'(0) > 0$ which implies $q > 0$ for an optimal contract. ■

G Proof of lemma 2

Using $I(\ell) \geq 0$ allows us to write

$$EU(\ell, \hat{\ell}) \leq \beta(\hat{\ell})Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) - B) + [1 - \beta(\hat{\ell})]u(w - P - \ell + I(\hat{\ell}) + I(\ell))$$

if $\hat{\ell} > \ell$. Furthermore, using $\beta(\hat{\ell}) \geq \beta(\ell)$ if $\hat{\ell} > \ell$ implies

$$EU(\ell) \geq \beta(\hat{\ell})Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) + [1 - \beta(\hat{\ell})]u(w - P - \ell + I(\ell)).$$

Hence, a sufficient condition for $EU(\ell) \geq EU(\ell, \hat{\ell})$ to be satisfied is written as

$$\begin{aligned} &\beta(\hat{\ell})\{Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) - Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) - B)\} \\ &\geq [1 - \beta(\hat{\ell})]\{u(w - P - \ell + I(\hat{\ell}) + I(\ell)) - u(w - P - \ell + I(\ell))\}. \end{aligned}$$

We have

$$\begin{aligned}
& Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell)) - Eu(w - P - \ell + I((1 - z(\tilde{x}))\ell) - B) \\
&= \int_0^B \int_0^1 u'(w - P - \ell + I((1 - z(\tilde{x}))\ell) - b) dG(x) db \\
&\geq \int_0^B u'(w - P - \ell + I(\ell) - b) db \\
&= u(w - P - \ell + I(\ell)) - u(w - P - \ell + I(\ell) - B)
\end{aligned}$$

Since $I(\cdot)$ is non-decreasing, we have $u'(w - P - \ell + I(\ell) - b) \leq u'(w - P - \ell + I((1 - z(x))\ell) - b)$ for all $x \in [0, 1]$. Consequently, a sufficient condition for $EU(\ell) \geq EU(\ell, \hat{\ell})$ is given by

$$u(w - P - \ell + I(\ell)) \geq \beta(\hat{\ell})\{u(w - P - \ell + I(\ell) - B)\} + [1 - \beta(\hat{\ell})]u(w - P - \ell + I(\ell) + I(\hat{\ell})).$$

If (7) is satisfied for all $\hat{\ell} \in (0, L]$, then individuals with wealth $w - P$ do not benefit from choosing one of the lotteries $Z(\hat{\ell}) \equiv (-B, \beta(\hat{\ell}); I(\hat{\ell}), 1 - \beta(\hat{\ell}))$, $\hat{\ell} \in (0, L]$. Thus, using $I(\ell) \leq \ell$, under NIARA preferences it is also true for individuals with wealth $w - P - \ell + I(\ell) \leq w - P$, and consequently if (7) holds we have $EU(\ell) \geq EU(\ell, \hat{\ell})$ for all $\ell \in (0, \hat{\ell})$. ■

H Proof of proposition 6

Denote by

$$\mathcal{L}(I(\cdot), P, z(\cdot), \lambda) \equiv Eu^*(I(\tilde{\ell}), P, z(\tilde{x})) + \lambda\{P - \pi E[I(\ell)I((1 - z(\tilde{x}))\ell)/(I(\tilde{\ell}) - c\tilde{\ell})]\}$$

the Lagrangian of program (9) neglecting the constraint on $I(\tilde{\ell})$. Suppose that $z(x) = 0$ for all $x \in [0, 1]$ at an optimal solution. The first-order optimality condition for P simplifies to

$$\partial\mathcal{L}/\partial P = -(1 - \pi)u'(w - P) - \pi Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) + \lambda = 0. \quad (16)$$

We deduce from the concavity of $u(\cdot)$ and $I(\ell) \leq \ell$ for all ℓ that $Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) \geq u'(w - P)$ and (16) implies $Eu'(w - P - \tilde{\ell} + I(\tilde{\ell})) \geq \lambda$ with a strong inequality if $I(\ell) < \ell$ on a positive measure subset of $(0, L]$. Furthermore, a point-wise maximization gives

$$\partial\mathcal{L}/\partial I(\ell) = \pi f(\ell)\{u'(w - P - \ell + I(\ell)) - \lambda[I(\ell) - 2c\ell]I(\ell)/[I(\ell) - c\ell]^2\} \geq 0 \quad (17)$$

for all ℓ , with an equality if $I(\ell) < \ell$. As $[I(\ell) - 2c\ell]I(\ell) < [I(\ell) - c\ell]^2$, we would have

$$u'(w - P - \ell + I(\ell)) < \lambda \leq Eu'(w - P - \tilde{\ell} + I(\tilde{\ell}))$$

for all ℓ such that $I(\ell) < \ell$ which is impossible. We thus have $I(\ell) = \ell$ for all $\ell \in (0, L]$. Regarding the nitpicking strategy, we have

$$\frac{\partial \mathcal{L}}{\partial z(x)} = -\pi g(x) E \left[\tilde{\ell} I'((1 - z(x))\tilde{\ell}) \left[\beta(\tilde{\ell}) u'(w - P - \tilde{\ell} + I((1 - z(x))\tilde{\ell})) - \frac{\lambda I(\tilde{\ell})}{I(\tilde{\ell}) - c\tilde{\ell}} \right] \right].$$

Using $z(x) = 0, I(\ell) = \ell$ and $\lambda = u'(w - P)$ gives

$$\frac{\partial \mathcal{L}}{\partial z(x)} > \frac{\pi c \lambda g(x)}{1 - c}$$

for all x such that $g(x) > 0$, which contradicts $z(x) \equiv 0$. ■

I Proof of proposition 7

Consider the optimal contract $\{I, P, z(\cdot)\}$ characterized in Section 2.2 under the assumption $e = 1$. Let Z_e be defined by

$$u(w - P - L + (1 - Z_e)I) \equiv \int_0^1 u(w - P - L + (1 - z(x))I) dG_e(x)$$

As G_0 FOSD G_1 , we have $Z_0 > Z_1$. If $d_0 = d_1 = d$ and $\pi_0 = \pi_1 = \pi$ then

$$Eu_1^* - Eu_0^* = \pi \beta^*(I, P) [u(w - P - L + (1 - Z_1)I)] - u(w - P - L + (1 - Z_0)I) > 0.$$

Consequently, there exist $\varepsilon, \varepsilon' > 0$ such that $Eu_1^* - d_1 > Eu_0^* - d_0$ if $d_1 - d_0 < \varepsilon$ and $\pi_0 - \pi_1 < \varepsilon'$. ■